85th Western Economic Association Conference
29 June-3 July 2010 in Portland

GOSSEN EQUATION HISTORY TO THE 2011 UNIFICATION WITH NEOCLASSICAL MICRO-ECONOMICS THEORY

By Thomas E. Chamberlain, Ph.D.*

But to see with my own eyes, and to hold in my hands, a great book, which had cost its author years of meditation and study and which had almost fallen into eternal oblivion—for this I was not prepared. Excerpt of Walras' tribute to Gossen—from "Walras on Gossen" 1885 (1952).

ABSTRACT

Socioeconomic stability with receding international conflict is closer at hand due to two modern developments: (1) A general recognition and acceptance of the permanent nuclear détente between major powers, allowing international cooperation for the common good; and (2) The deepening of neoclassical economics to its psychological foundation, thereby removing a barrier to sustained progress. A principal result in the latter development is the Gossen equation, a mathematical formulation in psychology representing the individual's expectational (intertemporal) plan and his or her psychosomatic feeling-state in anticipation of this plan, accounting for uncertainty/risk—a formulation in progress since Hermann Gossen's (solitary) 1854 contribution, and completed by the writer in 1993. In 2011 the Gossennian approach to mathematical economics was united with Leon Walras' and W. Stanley Jevons' approach (the foundation for neoclassical economics) thereby resolving a 135+ year schism or divide in the subject. With this unification, several advances derived of applied mathematical economics on the Gossenian side—such as periodic micro/macro-economic function based on nested-characteristic-times, the risk-versus-marginal productivity relation, completion of the Walrasian input/output substitution relations, and the Discretionary Power Principle of Justice—are carried over to the neoclassical side. …An overarching history of the Gossen Equation is provided, with additional emphasis on the author's theoretical contributions to the equation in the early 1990s along with his application of the equation in developmental and welfare economics in the early 2000s. The principal milestones of this history are given in a timeline chart (Figure 1) and a description of the Gossen equation is provided in a second chart (Figure 2). As an appendix to the paper, the written critique offered by a paper-discussant at the WEAI Conference in Portland (2010) is provided, with responses to her comments and questions. Additionally, the recent (March 2011) unification of neoclassical and Gossenian mathematical economics at the utility foundation (by way of a new "duration-for-consumption" constraint on the commodity utility function) is noted at several points in the article. This unification, in turn, admits a possible resolution of the long-standing division between the Austrian and neoclassical traditions.

This paper was originally completed in early 2010 and presented at the IAES conference in Prague (2010 March), and later at the WEAI conference in Portland, Oregon (2010 July). At the time Gossenian theory was a stand-alone approach to mathematical economics (dating from 1854), as developed by others over the decades and recently by the writer. But a new paper in early 2011 integrated Gossenian and neoclassical mathematical economics (See http://ssrn.com/abstract=1798772**). References to this unification of mathematical theory after the 135-year divide are given in the present updated paper.**

* Independent Researcher. Los Angeles, CA. Rev 5a; January 21, 2012.
tomchamb@ix.netcom.com

INTRODUCTION

Just after I presented a paper on Gossenian mathematical economics at the 2000 January Pacific Rim conference in Sydney, Australia, a listener who was a quite recent MIT PhD graduate in economics allowed that he had not heard of Hermann Gossen—a major contributor to mathematical economics who discovered the principles of marginal theory some 20 years before the contributions of W. Stanley Jevons (1871), Carl Menger (1871), and Leon Walras (1874-77) in the marginal revolution of the early 1870s. This apparent limitation of the economics curriculum may be judged even more remarkable inasmuch as Walras, believed by many to be the father of modern economics, considered Gossen "one of the greatest of economists that ever lived." In his writing following discovery of Gossen's book (in 1878) Jevons provided a similarly high opinion.

An important question concerns why significant aspects of Gossen's theory have been ignored in standard mathematical economics over the 133 years since his 1854 book resurfaced in 1878—and was praised by both Jevons and Walras. One answer is that the fateful misstep during the Marginal Revolution of the 1870s of identifying utility directly with consumable goods—without qualification or substantive explanation—set the academic discipline on the wrong course, where some of Gossen's deeper insights[1] were irrelevant or of no use (see Nicholas Georgescu-Roegen's introduction to Gossen's book (1983)). After all, when duration-for-consumption is ignored in economic modeling (typically, in neoclassical theory), or considered fixed when it is recognized in the individual's utility calculus, the time constraint, as a limitation or restriction in modeling the "business of life," is undermined. (Modern technological civilization would be placed in jeopardy—to put it mildly—were physicists and engineers to similarly discard the principles (laws) of thermodynamics.)[2]

A second reason why Gossen has been overlooked by mainstream economists is that his approach may be deemed too complicated. Prominent economists have maintained that economic systems cannot be understood or analyzed in an approach similar to that employed in physics and engineering. As an example, Kenneth Galbraith, in an aside while addressing why economists remain fixed to the classical and neoclassical traditions, observed that "… the reality of economic life … is not, in its varied disorder, suitable for mathematical replication." (1987, pg 285.) …It should be noted in response that physicists and engineers do not exactly model the subjects of analysis—they necessarily make simplifying

[1] Most relevant, in this regard, was Gossen's recognition of "recurrence of wants" (see [1854] 1983, p. xxx), which is certainly necessary in the complete formulation of the individual's intertemporal planning.

[2] It may be noted here that a more recent paper by the writer, "Fully temporal system linking productivity to risk and yielding completed input/output substitution" (2011), has provided a mathematical framework that accommodates *variable* duration-for-consumption thereby rendering the standard (neoclassical) paradigm fully temporal. This new framework serves to unite neoclassical and Gossenian theory.

abstractions and assumptions in order to arrive at determinate and useful solutions. In view of the recent success of the Gossenian approach in providing an explanation of runaway inequality and recommending corrective governance measures, it could be concluded that economists have been overly modest or reserved in their objectives.

Progress in understanding the function (and dysfunction) of economic behavior is not blocked by complexity as some economists have suggested. While real-world economics is indeed complex, much of the difficulty has yielded to investigation along an alternative path missed 140 years ago.[3] The primary intent of the present article is to trace this path from Gossen's contribution in 1854 through completion of the Gossen equation in 1993 and its unification with neoclassical economics in 2011. Subsequent progress in understanding and arresting runaway inequality (and poverty) based on the Gossenian approach is also given, along with two principal overview papers presented in Berlin and Beijing. …For additional information on Hermann Gossen's life and scholarly work the reader is referred to Georgescu-Roegen's introduction to the English translation of Gossen's book.

GOSSEN EQUATION MILESTONES

Figure 1 provides a timeline of the evolution of the Gossen equation starting with Gossen's original 1854 contribution and ending with completion of his equation by the writer in 1993. Also provided are follow-on milestones representing advances in developmental economics—based on the now-completed Gossen equation. Unification of the Gossen equation with neoclassical mathematical theory concludes the milestone chart.

An overview of the Gossen equation is given in Figure 2.[4] It is seen in the figure that the individual's expectational plan (the plan actually being followed, or the plan only under consideration) is comprised almost entirely of all terms to the right of "$\underline{\mathbf{F}^{\mathbf{i}} \equiv}$ " along with the constraints $\mathbf{\Phi}^{\mathbf{ic}}_{\mathbf{w}}$ on the equation itself. The left-hand-side of the equation (i.e., "$\underline{\mathbf{F}^{\mathbf{i}} \equiv}$ ") represents the individual's *psychosomatic response* to his or her anticipation of the plan-of-action expectationally considered (see Shackle, 1958, and Damasio, 1994). His or her (psychosomatic) anticipation accounts for expected risk (entering via the *worldline occurrence probabilities*, $\mathbf{p}^{\mathbf{i}}_{\mathbf{w}}$) along with *essential discounting* $\boldsymbol{\lambda}^{\mathbf{i}}_{\mathbf{w}}$ of *expected experience* (feeling state) $\mathbf{F}^{\mathbf{i}}_{\mathbf{w}}$**.** Of the several plans resolutely and decisively considered, that plan which provides the

[3] In this regard, one of the paper discussants at the IAES/Prague conference commented that epistemological constraints or guidelines have been, and continue to be, dismissed by prominent schools of economic thought. However, even if the neuropsychological (empirical) foundation for mathematical economics is dismissed as irrelevant, there remain the issues of mathematical completeness and coherence of the essential formulation and the overarching analytical framework built thereon. Here, at a minimum, one may properly conclude that a false basis of any theory can be misleading.

[4] The reader is referred to the 2003c paper for a substantive development of the Gossen equation.

least negative, or highest positive, feeling-state is selected. With the passage of real-time, plans are continually refined (i.e., due to uncertainty extinguished) with the receipt of expected information. But the plans are frequently modified (however slightly) when the individual receives surprising information—and they are sometimes replaced by a profoundly different plan when the individual receives surprising information requiring immediate and substantial redirection.

As noted in the figure, "The Gossen equation represents an individual's planned activity in general. The equation becomes economics-specific when economic activity (both productive and consumptive) is modeled along with definition of the corresponding economic constraints."

Returning now to the milestone chart (Figure 1), it is seen to be divided into three phases: **Discovery** (…of Gossen's book); **Development** (…of the Gossen equation); and **Application** (…of the Gossen equation to the Economic Problem [poverty and unbounded inequality] and formulation of a new developmental economics paradigm). As a preliminary step, each of the three phases is briefly discussed below.

Discovery. As seen in the timeline figure, Jevons was first to learn of Gossen's book, in 1878, several years after the Marginal Revolution had been completed and documented, including the mistaken (and clearly misleading) "direct identification" postulate. Just before crossing into the next century Walras (1896) appears to have admitted or acknowledged that Gossen was correct after all in postulating activity-based utility. But it was too late: The relatively simple (sometimes called 'simplistic') assignment of utility directly to economic goods and services—rather than formally recognize that utility is properly *imputed* to all entities, regardless of whether productive or consumptive; private or public; personal or shared, etc.—had gained the acceptance of leading mathematical economists.

Development. This phase of the timeline begins with Walras' reference to Gossen's work in 1896, and ends with completion of the Gossen equation in 1993. Along the way, advances that directly or indirectly contributed to the equation, or provided useful mathematical aids in application, are indicated. Noteworthy publications, such as Hicks (logical, but ill-founded) advocacy of ordinal utility in 1939, are also noted.[5]

[5] At the IAES conference in Prague, questions and discussions by the audience turned to the ordinal versus cardinal utility issue, still controversial decades after Hicks' "proof" that utility must be ordinal. An attendee noted that because risk is germane in decisions, utility cardinality must prevail in real-world expectation and experience—a conclusion (noted in the discussion) that was analytically demonstrated by Leontief (1966).

FIGURE 1. GOSSEN EQUATION---DISCOVERY, DEVELOPMENT AND APPLICATION MILESTONES

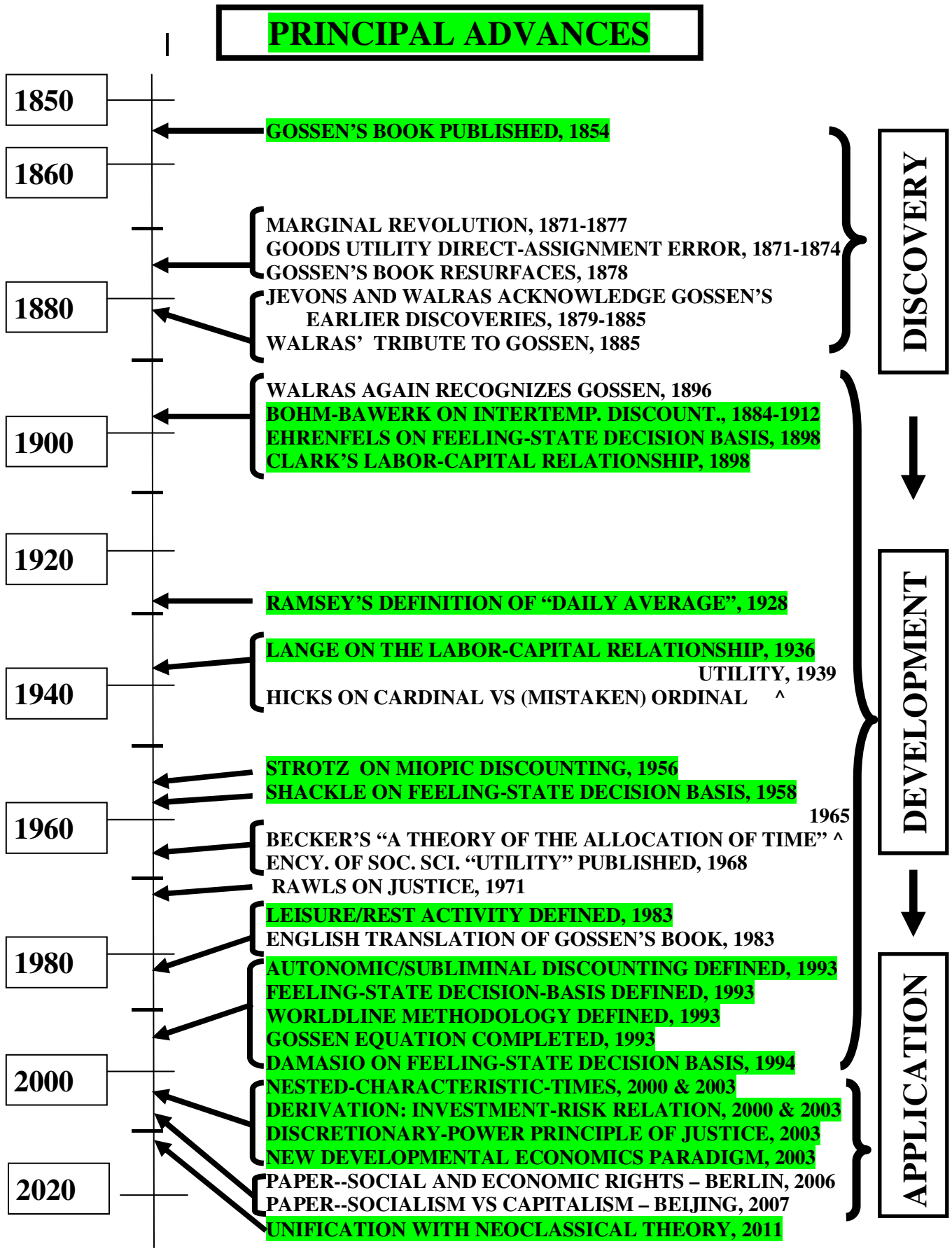
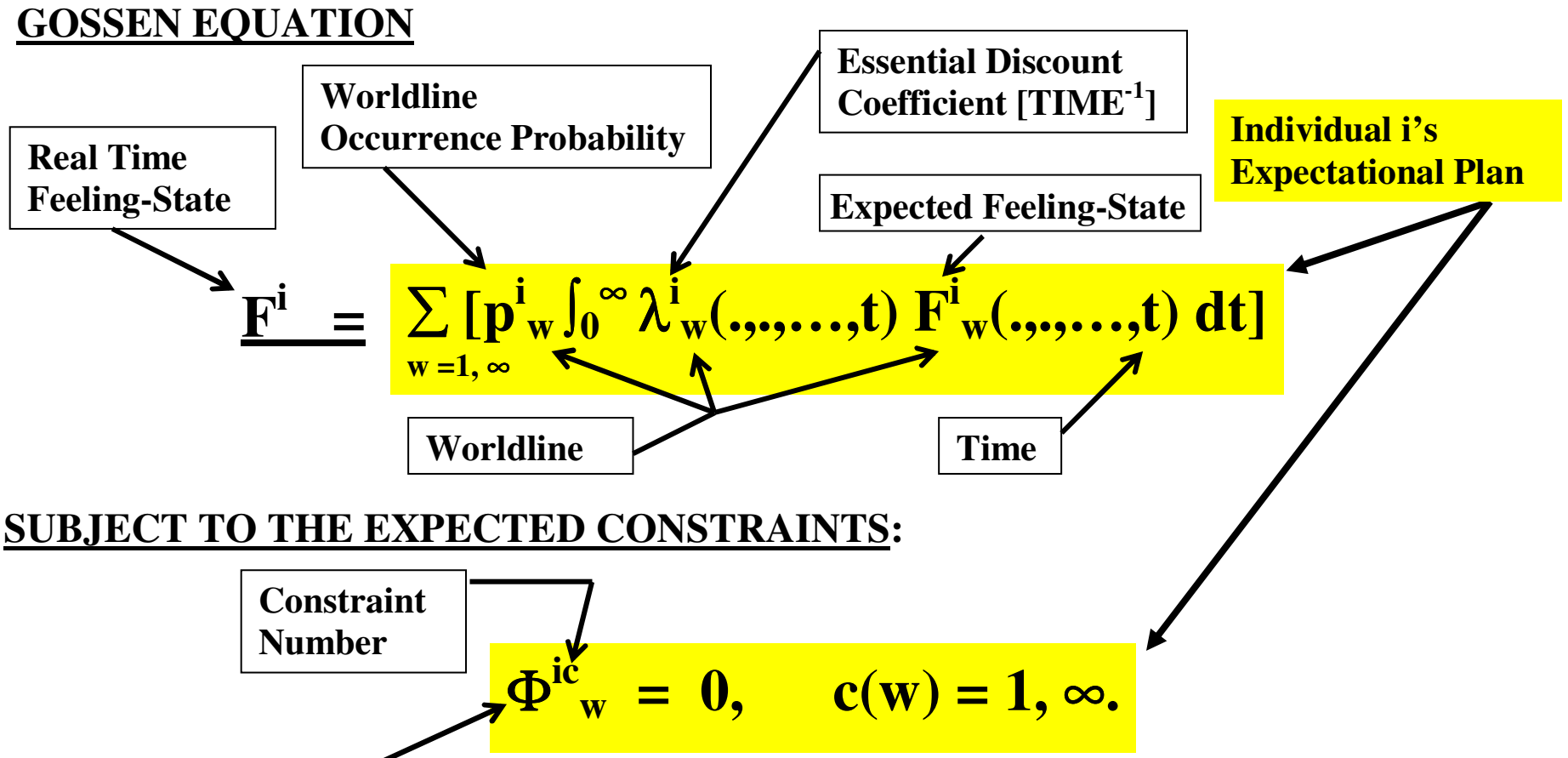


FIGURE 2. GOSSEN EQUATION OVERVIEW +



+ The Gossen equation represents an individual's planned activity in general. The equation becomes economics-specific when economic activity (both productive and consumptive) is modeled along with definition of the corresponding economic constraints.

Application. Although application of the Gossen equation began with investigation of social psychology effects (in the paper "On the psychological basis of economics and social psychology," completed in 1998) along with mathematical modeling in several microeconomic studies (not shown in the figure), it was with the 2003 paper "Does uneven expected risk promote poverty and instability?" that the study of inequality/poverty and economic development was initiated, with specific recommendations or suggestions.[6] Of the eleven follow-on papers to the present date, six have addressed poverty and developmental economics making use of the analytic and conceptual results of the "uneven expected risk" contribution. The two most favorably received of the six, on the basis of downloads from the Social Science Research Network, were "Relationship of economic stability to social and economic rights" (2006/7a) and "Socialism versus capitalism—economic stability as a unifying goal" (2006/7b).

DISCOVERY[7]

It can be beneficial or helpful in seeking progress in any scientific department to study our experience in other scientific departments. Here the rise of the human sciences (primarily psychology and economics [applied psychology][8]) over the past few centuries is similar to the rise of the natural sciences (primarily physics) over the past several millennia. As an example, our understanding of Earth's place in the cosmos experienced a transition from the conception of the ancient Greeks through the Renaissance. Aristarchus, of the early philosophers, was alone in advocating the sun-centered model of the planetary system. But his concept was ultimately confirmed during two centuries of investigation and study by Copernicus, Brahae, Kepler, Galileo, and Newton.

Fateful Misstep. The eventual acceptance of Aristarchus' heliocentric model may have its counterpart in the eventual acceptance of Hermann Gossen's human-activity based approach to mathematical economics—as aided or hastened by its recent (mathematical) unification with neoclassical theory.

---

[6] The relationship of marginal productivities (of direct labor, indirect labor, and capital) to expected risk was originally formulated in the 2000 paper "On the role of subjective uncertainty in the business cycle", with attention first given to poverty and development in the cited 2003 paper.

[7] This section is partly transcribed from the 2009 paper "World Bank Growth Report—assessment and extension".

[8] It is clear that economics has a psychological dimension, where, for example, expectation is profoundly psychological in nature. In this regard, expectation is integral with economic planning, and thereby affects market function and pricing. Then, in the same sense that aerodynamics is applied physics (in the goal, for example, of designing aircraft), and meteorology is applied physics (in the goal of predicting weather), so economics is applied psychology in the goal of understanding, predicting, and stabilizing economic activity.

While working in isolation from the academic community, Gossen established the essential foundation for utility (human satisfaction) in mathematical behavior[9]—that is, he recognized and postulated that subjective utility is properly identified only with *human activity* in canonical or fundamental theory. This means that utility must be exclusively identified with the individual's productive activity, consumptive activity, and leisure/rest in basic economic theory, and only imputed to all other entities that may enter the individual's conscious consideration.

The principals of the Marginal Revolution (Jevons, Menger, and Walras) published their books almost simultaneously in the early-to-mid 1870s, without knowledge of Gossen's deeper theory published 20 years earlier. Their scholarly direction was (in effect) to accommodate psychology in economic theory.[10] In so doing, early nineteenth century Classical Economics, which explained (natural) prices as proportional to the summation of labor value contributions, crossed the threshold into neoclassical economics that we've preserved and developed over the past 135+ years.[11] However, because the principals of this revolution in mathematical economics did not pay close attention to coherence and completeness in formulating basic (utility) theory, a crucial misstep occurred at the foundation—in particular, utility was directly assigned to commodities and services rather than exclusively (in basic theory) to human mental and physical activity.

Discovery of Gossen's Contribution. Just before publishing his work on marginal economics in 1874, Walras was understandably disappointed when Jevons wrote to him in late 1873 advising that the discoveries had been presented in his own book in 1871. The two men then properly and responsibly began an exhaustive search to find earlier writers who had made the discoveries. They initially found none and documented their conclusion in an article published in December 1878. However, after article acceptance and just before its publication, surprise once again intervened when Jevons became aware of Gossen's book, in August 1878. Walras received his own copy for review five months later in January 1879. He agreed with Jevons that Gossen had formulated the essential principles over twenty years

[9] Human satisfaction (utility) is, formally, time-integrated feeling-state, where measurable feeling state (the same as "instant utility") is the (scientifically) essential parameter. To briefly recap, utility and instant utility enter the individual's expectational plan by way of his or her (expected) activities (of every kind) and become imputed to (expected) entities of every kind. The resulting (expected) marginal utilities, activity-based and imputed, are crucial to economic function and behavior across the board (including, as examples, market pricing; rate of consumption; rate of saving; and capital growth and function).

[10] Profound insights are not always fully understood by their originators. Were Walras and Jevons mindful, in this regard, of introducing psychology into economics? In any case, a turn in this direction (albeit hesitant over the decades) was provided by their contributions.

[11] While Marshall is frequently credited with originating neoclassical theory, it appears that the marginalist's (albeit partial or incomplete) attention to psychology in economic behavior of the individual comprised the pivotal turn from classical economics to modern (mathematical) theory.

earlier. The discovery and its assessment were addressed in later editions of their books, and in Walras' 1885 tribute to Gossen.

As noted earlier both Jevons and Walras were impressed by Gossen's contribution. Jevons referred to Gossen's fundamental theory as "more general and thorough", and Walras' high opinion is evident in the quote on the title page. But while the two men had knowledge of Gossen's more consistent utility theory—in particular, to reiterate, utility was identified exclusively with human activity (mental and physical) in basic theory—in new editions of their books they overlooked Gossen's duration-for-consumption as a salient variable and continued to identify utility directly with commodities (products and services).[12] Later economists proceeded from the formal (and original) works of Jevons and Walras—and developed the neoclassical economics paradigm that has survived and evolved to the present day.

DEVELOPMENT

Had the three principals of the marginalist revolution known of Gossen's book—and had Walras, in particular, been thus acquainted—they could have seriously questioned the identification of utility with commodities in core or basic theory. They might have accepted and promoted Gossen's idea that activity-based utility (time-integrated feeling-state) should be exclusively and coherently identified with the individual's physical and mental activity. On the basis of this essential idea, researchers over the years and decades into the twentieth century would have approached social and economic matters differently, with different results, however large or small they may have been.

But Gossen's contribution has been largely ignored in mainstream economics throughout the twentieth century and into the new millennium. Yes, it is true that researchers have advanced aspects his theory (albeit typically without realizing or acknowledging Gossen's original contribution). However, discussion and development of the essential ideas have been minimal in the literature (of these, certainly the most prominent is Georgescu-Roegen's introduction to the English translation of Gossen's book). This lack of interest within the academic and publishing communities may not change in the near future—notwithstanding the danger from growing inequality and financial imbalances, and the very recent unification of Gossenian and neoclassical economic theory. As a present-time substitute, an overview of the evolution of the Gossenian theory is provided below, with emphasis on the writer's contribution in the early 1990s.

[12] It was perhaps too great a challenge for the two scholars to do otherwise given their lack of mathematical expertise (as historians have indicated regarding Walras (see Jaffe 1973, p.133) and Jevons acknowledged (p. xxxv)).

Development of the Gossen Equation. In essence the Gossen equation (with reference to the overview in Figure 2) resides in the domain of basic psychology, in much the same sense that Einstein's field equations of general relativity or Schrodinger's equation of quantum mechanics reside in the domain of basic physics.[13] Gossen used his essential concepts in human behavior to formulate principles or laws of human economic behavior. His accomplishments in economics comprise applied psychology (or applied human behavior) in a manner similar to the way meteorology is applied physics. And because the Gossen equation is an essential formulation of psychology—and resides at a deeper level than economics—it enables a more rigorous explanation and analysis of economic dysfunction (e.g., increasing inequality and poverty) and thereby reveals candidate solutions.

This psychological basis for Gossen's economics could partly explain the reluctance of the economics community to accept his ideas. In this regard, psychologists may be uneasy with mathematical formulation and investigation, as, in earlier times, substantive investigation of the natural world was discouraged (certainly an understatement). In any case, 115 years elapsed between the discovery of Gossen's book in 1878 and completion of the equation by the writer in 1993. Along the way Gossen's ideas were advanced by prominent economists (again, many without citing Gossen's book)—with Nicholas Georgescu-Roegen being the most noteworthy in his scholarly advocacy of Gossen's contributions.

As noted in the preceding section and indicated in Figure 1, an English translation of Gossen's 1854 book wasn't published until 1983. Georgescu-Roegen wrote the introduction, and therein provided a history of Gossen's life followed by a review of his mathematical theory. Georgescu-Roegen continued by recounting or discussing the scholarly references to Gossen's contribution over the decades into the twentieth century. He contributed to development of the Gossen equation by adding rest/leisure to the labor and consumption activities addressed in the original book. In addition, he may be considered the first to criticize neoclassical economics as epistemologically unsound—by pointing out that the discipline overlooked empirically measurable instant utility (feeling state) in favor of non-essential utility (assigned directly to commodities) at the basic or foundational level.

[13] To be more precise, the Gossen equation and constraints represent or reflect or model the individual's expectational plan, including the expected feeling of planned activity and the real time feeling-state in consideration and anticipation of this plan. In its essential (mathematical) formulation, the equation and the associated constraints are exclusive of economic modeling. It is only when production and consumption are introduced, as explicit parameters in the Gossen equation and applied constraints, that the formulation acquires an economic tenor or character.

In 1993 autonomic or subliminal discounting over the individual's intertemporal (expectational) plan was inserted into the equation.[14] In the same year the worldline concept—an invention bearing some similarity to the world line of physics—was applied along with its associated occurrence probability. This formulation assimilated or combined the contributions of numerous scholars (Ehrenfels (1896), Shackle (1958), Strotz (1956), and, as noted, Georgescu-Roegen, to name a few). The formulation was completed by the two above-mentioned contributions, and the equation has not been conceptually or mathematically adjusted over the years to the present time.

APPLICATION

In the years up to 2003 a question sometimes emerged whether the Gossenian approach could have any practical significance. An issue in this regard was the mathematical complexity of the approach—pointed out by a discussant at an economics conference in Europe about ten years past. This "roadblock" was resolved by a combination of my background in applied mathematics in conjunction with key ideas from mainstream economics.

Regarding my background in applied mathematics, the aerodynamic equations of aircraft flight, including boundary layer theory, are quite complex and before computers considerable ingenuity was required in design and performance analyses. This frequently involved simplifying the governing differential equations depending on location within the flow field. One simplifying approach was to place the equations in non-dimensional form followed by dropping the revealed or discovered negligible terms in the corresponding parts of the flow field. A solution throughout the aircraft flow field was then obtained by patching the separate solutions together.

This kind of simplification has been extensively applied in aerodynamics and fluid mechanics, and similar approaches have been employed in economics—Alfred Marshall's *negligibility of indirect effects* (1890) being an example. But now, due to the more essential character of the Gossenian approach, new opportunities have emerged to transform or simplify the modeling of challenging economic problems. A case in point is the long-standing or unsolved problem of relating capital productivity to labor productivity under expected (investment) risk.[15] Application of the deeper Gossenian approach has

[14] This parameter (λ) in the Gossen equation serves a second function (in addition to subliminal discounting) by converting intertemporal utility into real-time psychosomatic feeling-state (which was given empirical support the human-decision studies conducted by Damasio (1994)). In this regard, a paper-discussant at the IAES-Prague conference asked whether the Gossen equation could model or represent "regret" in the individual's expectational plan. The response was that regret can be represented or reflected, as may (possibly) all emotions in their psychosomatic dimensions. Furthermore, in response to another question, the Gossen equation can be tested, and possibly disproved, in various empirical studies using human and animal subjects. (See Appendix A.)

[15] Solow (1965) observed that analysis of capital function is "very complicated and very difficult," and that "..there is a further fundamental difficulty that bedevils even uncomplicated models." In particular, "Capital problems are

yielded a functional expression for this relationship,[16] which then allowed insight into a cause of the Economic Problem (i.e., of runaway inequality and poverty).

In this task which, again, investigated the effect of investment risk (an aspect of uncertainty) on the capital-labor relationship, the simplifying assumption consisted of dividing the intertemporal period of an entire economic system (of cooperating/coordinating individuals) into three overlapping or "nested" sub-periods with greatly different characteristic times (or intervals): (1) the 24-hour day; (2) the significantly greater characteristic time of investment-risk discounting; and (3) the still much greater characteristic time of macroeconomic change (referred to as "NESTED CHARACTERISTIC TIMES" in Figure 1). Modeling was accordingly permitted of the steady-state (equilibrium) planning of each individual in a (very slowly) evolving economy that is subject to a sudden shift in (expectational) investment risk—with a corresponding shift in the employment of labor— after which the economy continues to slowly evolve. (The assumption was first employed in the 2000 paper "On the role of subjective uncertainty in the business cycle", and three years later in "Does uneven expected risk promote poverty and instability?".)

A key idea from mainstream economics, in this regard, was Frank Ramsey's (albeit implicit) time-averaging of any given parameter (for example, averaging the instant utility of labor over a 24-hour day). This idea enabled (in my work) the conversion of discontinuous or intermittent parameters (like the feeling-state of labor, which is typically finite or non-zero only during certain intervals within each day) into continuous parameters throughout the day and to the intertemporal horizon, for the special but useful case where the characteristic times of expectational discounting and the economic system are both much greater than the 24-hour day.

These ideas served to greatly increase the analytical power of Clark's (1899) and Lange's (1936) conceptualization of capital function (relating the productive power of capital to the application of indirect and direct labor) by extending its use or employment from the intertemporal single day to the multi-day period. It was the combination of these simplifications and extensions, and others as well, that led to policy prescriptions for arresting runaway-inequality and poverty (over time), and, as a closely related result, to the Discretionary-Power Principle of Justice.

Discretionary Power Principle of Justice. It could be understood that the principal conclusion or end-result of the effort on the Gossenian approach—involving: (1) demonstration of the unsound character of mainstream economics; (2) completion of the Gossen equation; and (3) application of this

---

inevitably bound up with questions of uncertainty, limited foresight, and reactions to the unexpected. He concluded "…without a satisfactory account of behavior under uncertainty we cannot have a complete theory of capital."

[16] Unification conveys this capability to neoclassical economics. In this regard, the relationship remains valid in the neoclassical limit of invariant duration-for-consumption.

equation to suggest policies for eliminating poverty and arresting runaway inequality—is the Discretionary Power Principle of Justice, a new rule for just or fair inter-relationships that permits the advantaged to increase their market-power and attending benefits provided that the "benefits and discretionary-power" of everyone else, and particularly the poorest of the poor, are improved. And what is the basis for this principle?

It is this: There is a natural tendency for the market economy to promote economic inequality (see 2003/4)—but this tendency involves not just the rich moving ahead faster than the rest of us, but the rich advancing while many of us fall in absolute terms, in the absence of support. Here we could tolerate growing inequality, and have for decades, with the assertion that the middle and the poor are at least moving ahead, however slowly. But the new analytic recognition that the poorest communities—with total population now over one billion—are in growing danger of collapse changes the calculus: Doing nothing to correct the fatefully growing inequality (where "you're on your own" government follows the individualist philosophy) must finally bring a general collapse.

The Discretionary-Power Principle is an extension of John Rawls' maxim or rule:

> "*…there is no injustice in the greater benefits earned by the few provided that the situation of persons not so fortunate is thereby improved*" (pg 15).

The difficulty with this maxim is that it does not specifically refer to the critical dynamic in human interrelationships addressed above. To reiterate, it does not explicitly or formally recognize the natural tendency of people of ordinary means—but the poor, in particular—to reduce or hold back investment in self, family, and community due to discretionary disadvantage in market competition with the ascending rich, a tendency that grows or magnifies as the wealthy pull further ahead in our free-wheeling markets. Eventually the capital intensity of the disadvantaged (i.e., in terms of their knowledge, skills, health, etc.) must fall as their investments fail to compensate for capital depletion and loss. Rawls' maxim is accordingly revised as follows (2003/4):

> "*…there is no injustice in the greater benefits earned by the few provided that the benefits and discretionary-power of persons not so fortunate are improved.*"

On this basis our progress in the free-enterprise capitalist system may proceed—but with continuing or never-ending policies and institutions that preserve and grow the capital intensity of ordinary people and the poor.

CONCLUSION

Hermann Gossen was understandably disappointed with the sales of his book on human behavior and economics in the several years following its publication in 1854, and he ordered a

recall of the remaining copies. Had he lived into the 1870s and 1880s his spirit may have revived from the high praise Walras and Jevons placed on his work following its discovery in 1878. But another 130 years would pass before Gossenian activity-based utility theory would converge with neoclassical commodity-based utility theory through unification of the closely-related, but nevertheless fundamentally dissimilar, mathematical approaches.

Neither Walras nor Jevons, with their books already in print, were positioned to reconcile Gossen's ideas with their own, this requiring a significant rewrite in both cases at a higher mathematical expertise than prevailed at the time. The incomplete neoclassical theory of economics emerging from the Marginal Revolution took hold, and has dominated the stage throughout the twentieth century up to the present day.

The Gossenian approach was not abandoned, however, and it gradually advanced through the research of both neoclassical and heterodox contributors. Of the latter, the late Nicholas Georgescu-Roegen was a prominent champion of Hermann Gossen's work, having written the comprehensive introduction to the English translation of his (Gossen's) book and extending the mathematical theory to account for leisure. In the early 1990s the present writer finalized the Gossen equation (in 1993) by bringing intertemporal discounting (including expected risk) and the feeling-state decision-basis into a coherent integral and summation formulation.

Eighteen years later, after a series of papers given at international conferences on both the positive and normative sides of economics, Gossenian and neoclassical mathematical (utility) theory were united (in early 2011). Besides restoring coherence and completeness within mathematical theory after a 135 year division, this unification introduces several Gossenian-derived normative recommendations into the neoclassical tradition and also may help resolve the long-standing divide between the Austrian and neoclassical philosophies of economic behavior.

APPENDIX

Professor Yang Wang's Discussion—

With the Author's Responses (As Underlined Entries)

History of the Gossen Equation and 21st Century Applications

Thomas E. Chamberlain, Independent Researcher
Discussant: Yang Wang, Lafayette College

Western Economic Association International
July 2010

Overview

- Gossen Equation – psychology-based alternative to neoclassical economic theory of utility and economic behavior.
- This paper provides overview of Gossen Equation in economic history and thought
  - Discovery
  - Development
  - Application

Key Differences from Neoclassical Economic Theory

- Neoclassical economic theory (Walras 1874-7, Marginal Revolution, Marshall…)
  - Utility is defined over consumable goods, instead of duration-for-consumption;
    - But—as Gossenian and neoclassical theory both agree—utility is also defined over duration-for-labor (Jevons, 1871) and duration-for-leisure (Walras, 1874-77).
    - Utility continued (rarely, however) to be assigned to productive activity. (For example, in Frank Ramsey's "A Mathematical Theory of Saving", 1928.)
  - Consumption activity is ignored
    - This deficiency of neoclassical theory has been prominently asserted by Nicholas Georgescu-Roegen (1983), and more recently by Ian Steedman (2001).
- Gossen Equation (Gossen 1854, Georgescu-Roegen 1983, Chamberlain 1997-2010) [The equation was originally completed in 1993.]
  - Utility is defined over *consumption activity*
    - Subjective utility—or, more fundamentally, instant utility (feeling state)—is also defined over *productive activity* and *leisure*.

- Other differences: measurability of utility–cardinal (not ordinal) utility
  - This is vital to economics as an analytic and explanatory tool—indeed, understanding capital-function, financial-instrument yields, interest, etc., requires cardinality. (Leontief has proved the importance of cardinality in accounting for investment-risk—1966, page 26.)

`

Contributions

- Fascinating discussion of the discovery and development of the Gossen Equation
- New derivation of the Gossen Equation from Gary Becker's "A theory of the allocation of time." (1965)[17]
- Applications of the Gossen Equation:
  - Discretionary Power Principle of Justice:

    "There is no injustice in the greater benefits earned by the few provided that the *benefits and discretionary power* of persons not so fortunate are improved."

[17] An assessment of Becker's 1965 theory of time in economics is now provided in "Fully temporal system linking productivity to risk and yielding completed input/output substitution" (2011).

► Additional applications include formulation of investment-risk versus labor/capital marginal productivities and completion of the neoclassical input/output substitution relations.

Comments and Suggestions

- Interesting and thought-provoking paper
- Some comments of theoretical and empirical nature ...

Comments and Suggestions

- Should we view it as THE paradigm of human behavior?
  - The theory is offered as an advance. In this regard, physics has not yet produced an all-encompassing theory or paradigm.

- Does it replace all the other frameworks, such as the neoclassical economic theory? Or is it an alternative/complementary to the neoclassical economic theory?
  - Gossenian theory may be considered an alternative to neoclassical mathematical theory. But now the two distinct mathematical formulations have been united (2011).

- Connection to behavioral economics and neuroeconomics?
  - Behavior economics is becoming a prominent "guiding light" in formulating U.S. policy to improve consumer decisions and help defeat poverty. But the objective and scope of the two research

areas are quite limited—in much the same sense that the equations-of-state (thermal and caloric) of fluid mechanics are limited in that they cannot, exclusive of the equations of motion, analyze and explain aerodynamics.

But the now unified (Gossenian and neoclassical) mathematical theory provides the necessary overarching formulation. Within the context of the unified theory, behavioral- and neuro-economics may contribute to economics modeling in a manner similar to how investigations of the equations-of-state of gases and liquids contribute to fluid mechanics modeling.

- What about macroeconomics? How to aggregate all the individual agents (and their feeling)?
  - In astrophysics we aggregate particle behavior to, for example, understand supernovas—indeed our progress here would be minimal without taking account of particle physics. Macro-physics is significantly based on, or derived from, micro-physics.

    Regarding macroeconomics, it is appropriate, indeed crucial, that we understand and formulate human interaction in modeling national economies. There is, for example, a natural tendency toward economic inequality in the market system, even when free

of all imperfections (i.e., moral hazard, discrimination, uneven innate capabilities, etc). This tendency should be recognized in the macro-models, if only as a "propensity term" which can be counteracted by appropriate policy and institutional measures.

Regarding feeling, it is important to note that feeling-state is essentially germane to the Gossen equation, in that both expected (intertemporal) feeling state and real time (anticipatory) feeling state are represented. But feeling state may be more appropriate to psychology than economics as a salient concept and parameter—for example, in the modeling of empathetic psychosomatic states and behavioral responses (probably in the distant future).

- Empirical evidence? How can the underlying assumptions be tested?
  - Nicholas Georgescu-Roegen advised that economic theory must be correct at its foundation:

> *Given that the only certain fact is the intensity of pleasure felt at an instant of time, the only epistemologically sound approach is to take intensity as the primary concept*. Introduction to the English translation of Hermann Gossen's book on economic behavioral theory ([1854] 1983, lxxxi).

This conclusion is supported by empirical measurements of instant utility (feeling-state) by Rolls (1975), Damasio (1994), and others.

- Gossenian mathematical theory—and now the unified theory as well—accommodates or "over-arches" the substantially literary Austrian Tradition, which accords with human economic behavior:

> *"[The Austrian] perspective is that which particularly emphasizes: the purposefulness of individual action; the role of knowledge in economic choice; the subjectivity of the phenomena that interest economists; and the ex ante role in which time affects activity."* (Kirzner 1981)

Additionally, Gossenian theory can explicitly model uncertainty and investment-risk. This new "standard model" of human economic behavior is empirically supported by extensive neuropsychological investigations of cognitive function (See page 11 of "Instant utility approach to the social sciences" in Chamberlain-West.com).

- Damasio's empirical studies [1994] of patients with impaired psychosomatic function point to anticipatory feeling-state as the basis for decisions. (But the neoclassical assumption that choice is based on maximized discounted intertemporal utility is also accommodated by the theory.)

This (Gossenian) choice theory can be empirically tested and possibly falsified. In this regard, choices in laboratory settings can be compared with neuropsychological and/or psychosomatic measurements at the same instant.

- Analytic corroboration of now implemented policies and institutions is another test. In this regard, application of the Gossen equation has identified uneven investment risk as a cause of growing inequality and "hollowing out" of the middle class (2003/4), and suggested two policy initiatives for stability and growth: (1) Conditional Cash Transfers to promote well-being through promotion of human-capital intensity (particularly of children); and (2) damping of international commercial/financial transfers. Measure (1), in particular, has shown success over the past decade in Brazil (and other Latin America countries).

REFERENCES

Antonelli, E. (1939). *L 'economie pure du capitalisme*. Paris, 1939.

Becker, G. S. (1965). A theory of the allocation of time. *The Economic Journal*, Vol. 75, No. 299.

Bohm-Bawerk (1884-1912). *Capital and interest.* 3 vols. South Holland, Ill.: Libertarian Press. First published as *Kapital und Kapitalzins.*

Brems, H. (1986). Pioneering economic theory: 1630-1980; A mathematical restatement. Baltimore: The John Hopkins University Press.

Chamberlain, T. E. (1997). On the psychological basis of economic behavior. Paper presented at the *72nd annual conference of the Western Economic Association International*, Seattle, Washington.

Chamberlain, T. E. (1998a). Mathematical formulation of the expectational plan. Paper presented at the *79th annual meeting of the Pacific Division, American Association for the Advancement of Science*, Logan, Utah.

Chamberlain, T. E. (1998b). On the psychological basis of economics and social psychology. Paper presented at the *38th Congress of the European Regional Science Association*, Vienna, Austria. (Available on the Social Science Research Network: http://ssrn.com/abstract=597341)

Chamberlain, T. E. (1999a). Effect of expectational planning on commodity prices and interest rates. Paper presented at the *47th Bi-Annual Conference of the International Atlantic Economic Society*, Vienna, Austria.

Chamberlain, T. E. (1999b). Small perturbation approach to a transient inter-regional economy accounting for wages, prices, and transaction costs. Paper presented at the *39th Congress of the European Regional Science Association*, Dublin, Ireland.

Chamberlain, T. E. (2000). On the role of subjective uncertainty in the business cycle. Paper presented at the *4th Pacific Rim Allied Organizations Conference*, Sydney, Australia. (Later given at the *40th Congress of the European Regional Science Association*, Vienna, Austria [2000].)

Chamberlain, T. E. (2001). Can economics be a true science? Paper presented at the *55th annual meeting of the Pacific Division, American Association for the Advancement of Science*, Irvine, CA.

Chamberlain, T. E. (2003/4). Does uneven expected risk promote poverty and instability? Paper presented at the *5th Pacific Rim Allied Organizations Conference*, Taipei, Taiwan. (Later given at IAES/Lisbon [2004], SASE/Washington [2004], and SABE/Philadelphia [2004].) (Available on the Social Science Research Network: http://ssrn.com/abstract=558045)

Chamberlain, T. E. (2003b). Mortality risk as a basis for time preference. Paper presented at the *55th International Atlantic Economic Conference,* Vienna, Austria.

Chamberlain, T. E. (2003c). Instant utility approach to the social sciences. Paper presented at the *annual meeting of the Society for the Advancement of Behavioral Economics*, Lake Tahoe, CA. (Available on the Social Science Research Network: http://ssrn.com/abstract=560421)

Chamberlain, T. E. (2005a). Neocapitalism and the reduction of global poverty. Paper presented at the *6th Pacific Rim Allied Organizations Conference*, Hong Kong, China. (Available on the Social Science Research Network: http://ssrn.com/abstract=652662)

Chamberlain, T. E. (2005b). Saving disparity and the United States current account deficit. Paper presented at the *60th International Atlantic Economic Conference,* New York. (Available on the Social Science Research Network: http://ssrn.com/abstract=834184)

Chamberlain, T. E. (2006/7a). Relationship of economic stability to social and economic rights. Paper presented at the *61st International Atlantic Economic Conference,* Berlin. (Later given at the *47th Congress of the European Regional Science Association*, Paris, France [2007].) (Available on the Social Science Research Network: http://ssrn.com/abstract=895000)

Chamberlain, T. E. (2006/7b). Socialism versus Capitalism—economic stability as a unifying goal. Paper presented at the *62nd International Atlantic Economic Conference,* Philadelphia. (Later given at the *7th Pacific Rim Allied Organizations Conference*, Beijing, China [2007].) (Available on the Social Science Research Network: http://ssrn.com/abstract=944337)

Chamberlain, T. E. (2007). Minimum wage function in relation to economic instability and poverty. Paper presented at the *64th International Atlantic Economic Conference,* Savannah, Georgia. (Available on the Social Science Research Network: http://ssrn.com/abstract=1079575)

Chamberlain, T. E. (2008). Economic and sectarian adjustments toward East-West normalization. Paper presented at the *65th International Atlantic Economic Conference,* Warsaw.

Chamberlain, T. E. (2009a). World Bank Growth Report—assessment and extension. Paper presented at the *67th International Atlantic Economic Conference,* Rome. (Later given at the *84th Western Economic Association Conference*, Vancouver [2009].) (Available on the Social Science Research Network: http://ssrn.com/abstract=1380656) (Title and text revised.)

Chamberlain, T. E. (2009b). In search of deeper understanding. *Financial Times* (letter), April 27, 2009.

Chamberlain, T. E. (2009c). True mathematical economics is vital. *Financial Times* (letter), December 18, 2009.

Chamberlain, T. E. (2010, 2011). Gossen equation history to the 2011 unification with neoclassical micro-economics theory. Paper presented at the *69th International Atlantic Economic Conference,* Prague. (Available on the Social Science Research Network: http://ssrn.com/abstract=1604948) (Title and text revised.)

Chamberlain, T. E. (2011). Fully temporal system linking productivity to risk and yielding completed input/output substitution. Paper presented at the *71st International Atlantic Economic Conference,* Rome. (Available on the Social Science Research Network: http://ssrn.com/abstract=1798772)

Clark, J. B. (1899). *The distribution of wealth*. New York.

Damasio, A. (1994). *Descartes' error: emotion, reason, and the human brain*. New York: G. T. Putnam's Sons.

Dolan, R. J. (2002). Emotion, cognition, and behavior. *Science*, Vol. 298, No. 6761, 8 November 2002.

Ehrenfels, C. von. (1896). Werttheorie. Vol. I: *Philosophische Schriften*, ed. R. Fabian, Munich: Philosophia (1982).

Galbraith, J. K. (1987). *Economics in perspective*. Boston: Houghton Mifflin Company.

Georgescu-Roegen, N. (1968). Utility. *International Encyclopedia of the Social Sciences*. David Sills (Ed.) The Macmillan Company and the Free Press.

Georgescu-Roegen, N. (1983). Introduction to Gossen's book. (See Gossen Ref.)

Gossen, H. H. (1854, 1983). *The laws of human relations and the rules of human action derived there from*. Translated by Rudolph C. Blitz with an introductory essay by Nicholas Georgescu-Roegen. Cambridge: MIT Press.

Hausman, D. M. (1992). *The inexact and separate science of economics*. Cambridge: Cambridge University Press.

Hicks, J. R. (1939). *Value and capital*. Oxford: Clarendon Press.

Jaffe, W. (1973). Leon Walras' role in the marginal revolution. From *The marginal revolution in economics: Interpretation and evaluation*; Durham, North Carolina: Duke University Press, Edited by R. D. Collison Black, A. W. Coats, and Craufurd D. W. Goodwin.

Jevons, W. S. (1871). *The theory of political economy*. New York: Kelley and Millman, Inc. (1957, 5th edition)

Lange, O. (1936). The place of interest in the theory of production. *Review of Economic Studies*, Vol. 3.

Leontief, W. (1966). *Essays in economics—theories and theorizing*. Volume One. White plains, N.Y.: International Arts and Sciences Press, Inc.

Marshall, A. (1890, 1948). *Principles of economics*. 8th ed. New York: Macmillan.

Menger, C. (1871, 1950). *Principles of economics: First general part*. Edited by James Dingwall and Bert F. Hoselitz, with an introduction by Frank H. Knight. Glencoe, Ill.: Free Press.

Pareto, V. (1906). *Manual of political economy*. 1971 translation of 1927 edition, New York: Augustus M. Kelley.

Ramsey, F. P. (1928). A mathematical theory of saving. *Economic Journal*. 38, 543-559.

Rawls, J. (1971). *A theory of justice*. Harvard University Press.

Rima, I. (1967,1991). *Development of economic analysis*. Boston, MA: Irwin.

Rolls, E. T. (1975). *The brain and reward*. Oxford: Permagon Press.

Shackle, G. L. S. (1958). *Time in economics*. Amsterdam: North-Holland Publishing Company.

Solow, R. M. (1965). *Capital theory and the rate of return*. Chicago: Rand McNally & Company.

Steedman, I. (2001). Consumption takes time: implications for economic theory. Routledge, London.

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies,* 23, 165-180.

Walras, L. (1874-1877). *Elements of pure economics: or, the theory of social wealth*. Translated by William Jaffe. Homewood, Ill.: Irwin: London: Allen & Unwin (1954).

Walras, L. (1885 [1952]). Walras on Gossen. From *The development of economic thought: Great economists in perspective*; New York: John Wiley & Sons, Inc., ed. H.W. Spiegel.

Walras, L. (1896 [1936]). *Etudes d'economic sociale*, 2nd Ed., Lausanne: Rouge.